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Nonlinear surface waves on a plasma sphere in an external electric field

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It is shown that nonlinear surface waves at the boundary between a spherical low-temperature plasma and a dielectric container can couple to constant or time-varying electric fields in the latter. Exact equations governing the interaction are obtained.

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There has been much recent interest in linear and nonlinear surface waves at the boundary of low-temperature plasmas (e.g., Refs. [1–10]). These modes are of importance in many applications of plasmas in modern technology, such as in plasma production and diagnostics, new sources of light, coherent radiation and particle beams, solid-state and optical devices, as well as in plasma-assisted material processing [2,4–8]. The surface waves can act as transmitters of energy and information along the interface as well as between the plasma and the confining medium. It is therefore of importance to understand the properties of the surface waves and their interaction with the volume waves within the plasma.

The existence of a class of exact solutions for finite-amplitude surface waves progagating on the boundary between a cold plasma and its dielectric container has been pointed out recently [9,10]. The solutions are exact in the sense that starting from the conservation equations for the cold plasma and Maxwell's equations, no approximations of any kind, such as series expansions or *ad hoc* truncations, need to be made in obtaining the eigenfunctions describing the space-time behavior of the wave motion. These solutions are of special interest since, besides describing the surface wave physics in a mathematically exact manner, they can also be used for verifying various approximation or numerical schemes in the study of nonlinear wave interactions and instabilities.

In this Brief Report, we generalize the investigation of Ref. [10] to include the effect of an external electric field on the nonlinear surface waves on a spherical plasma. The spherical geometry is relevant to many industrial plasmas used for processing small-scale electronic materials. It can also serve as a model for nonlinear dust particles in the study of scattering and absorption of light from them [11]. By first finding the appropriate spatial behavior (of the field quantities) which allows for the separation of the time and space variables in the governing equations as well as the boundary conditions, we obtain a set of coupled nonlinear ordinary differential equations for the temporal behavior of the system. It is found that steady and unsteady external fields in the surrounding dielectric can modulate and resonantly amplify certain surface modes, while leaving the other modes unaffected. The latter do not grow, although they are nonlinearly coupled to the amplified surface mode.

We consider a spherical electron plasma in the region $0 < r < R$ with a positively charged immobile background of lattice or heavy ions. Collisions and other forms of dissipation are neglected. The plasma is bounded at $r = R$ by an infinitely large rigid linear dielectric of constant permittivity ϵ_d. A spatially constant external electric field $\mathbf{E}_0(t)$ is assumed to exist in the dielectric at large distances from the plasma sphere.

The evolution of the electrostatic potential φ, the electron density n_e, and fluid velocity $\mathbf{v}$ is governed by the continuity and cold-fluid momentum equations for the electrons, as well as the Poisson equation

$$\nabla^2\varphi = -\frac{q}{\epsilon_0}(n_e - n_0), \tag{1}$$

where n_0 is the background ion density. The equations are completed by the boundary conditions requiring the continuity of the potential φ and the radial component of the total current density

$$[qn_e v_r - \epsilon_0\partial_t\partial_r\varphi]_{r=R-0} = [-\epsilon_0\epsilon_d\partial_t\partial_r\varphi]_{r=R+0} \tag{2}$$

at the boundary $r = R$. Note that (2) is a generalization of the often-used condition of the continuity of $\epsilon_0\epsilon_d\partial_r\varphi$ across the interface (see Refs. [9] and [10] for details).

The approach we shall use is similar to that of Lorenz [12] who investigated nonlinear atmospheric waves and deterministic chaos by first separating the spatial variations in the governing equations from the temporal one. However, here we shall make no *ad hoc* truncation of the higher harmonics [9,10], nor any other approximation. Accordingly, for the spatial wave structure inside the plasma, we make the Ansätze

$$n_e = n(t), \tag{3}$$

$$\mathbf{v} = \frac{v_a}{R}x\hat{\mathbf{x}} + \frac{v_b}{R}y\hat{\mathbf{y}} + \frac{v_c}{R}z\hat{\mathbf{z}} + v_\alpha\hat{\mathbf{x}} + v_\beta\hat{\mathbf{y}} + v_\gamma\hat{\mathbf{z}}, \tag{4}$$

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$$\varphi_{r<R} = \frac{x^2 - r^2/3}{R^2}\varphi_a + \frac{y^2 - r^2/3}{R^2}\varphi_b + \frac{z^2 - r^2/3}{R^2}\varphi_c + \frac{x}{R}\varphi_\alpha + \frac{y}{R}\varphi_\beta + \frac{z}{R}\varphi_\gamma + \left(\frac{r^2}{R^2} - 1\right)\varphi_s, \quad (5)$$

and

$$\varphi_{r>R} = R^3\left[\frac{x^2 - r^2/3}{r^5}\varphi_a + \frac{y^2 - r^2/3}{r^5}\varphi_b + \frac{z^2 - r^2/3}{r^5}\varphi_c\right] + R^2\left[\frac{x}{r^3}(\varphi_\alpha + RE_{0x}) + \frac{y}{r^3}(\varphi_\beta + RE_{0y}) + \frac{z}{r^3}(\varphi_\gamma + RE_{0z})\right] - E_{0x}x - E_{0y}y - E_{0z}z, \quad (6)$$

where n, $v_i(i = a, b, c)$, $v_k(k = \alpha, \beta, \gamma)$, φ_i and φ_k are functions of time only. The imposed electric field $\mathbf{E}_0(t)$ $(= -\nabla\varphi_{r\gg R})$ can be constant or any function of time. The potential $\varphi_{r>R}$ in the dielectric satisfies the Laplace equation exactly and $\varphi_{r>R} = \varphi_{r<R}$ at the boundary $r = R$. Consistent with the cold-plasma approximation, the thickness (which is of the order of the Debye length) of the surface layer at the interface has been taken to be smaller than any other characteristic dimension in the problem. We also assume that no electrical nor chemical effects that can affect the surface region are present. It is of interest to point out that (3)–(6) are considerably more general than the corresponding equations for the case [10] without the external field.

As mentioned, the Ansätze (3)–(6) have been constructed such that the space and time dependences of the field variables can be separated. In fact, substituting (3)–(6) into (1) and (2) and the electron conservation equations, and equating the coefficients of the various (spatial) harmonics, one obtains

$$d_\tau N + N(V_a + V_b + V_c) = 0, \quad (7)$$

$$d_\tau \phi_i - \frac{1}{3\epsilon_d + 2}NV_i = 0, \quad (8)$$

$$d_\tau V_i + V_i^2 = -2\phi_i + \frac{2\epsilon_d}{3\epsilon_d + 2}(N - 1), \quad (9)$$

$$d_\tau \mathcal{E}_k = -\frac{1}{2\epsilon_d + 1}(3\epsilon_d d_\tau \mathcal{E}_{0j} - NV_k), \quad (10)$$

$$d_\tau V_k + V_i V_k = -\mathcal{E}_k, \quad (11)$$

where $N = n/n_0$, $j = x, y$ or z, and $\tau = \omega_p t$. We have also defined $V_{i,k} = v_{i,k}/R\omega_p$, $\phi_i = \epsilon_0\varphi_i/n_0qR^2$, $\mathcal{E}_k = \epsilon_0\varphi_k/n_0qR^2$, and $\mathcal{E}_{0j} = \epsilon_0E_{0j}/n_0qR$. When the indices i, j and/or k appear in the same equation, the ones that are similar in the alphabetical ordering are to be taken. In addition, the quantity φ_s appearing in (5) is given by $(1 - N)n_0qR^2/6\epsilon_0$.

Thus, we have effectively 13 unknowns, N, V_i, V_k, ϕ_i, and $\mathcal{E}_k$ governed by 13 simple nonlinear ordinary differential equations. In fact, the equations are in such a form that one can first solve for the seven variables N, V_i, and ϕ_i using the seven equations given by (7)–(9), and then use the six equations given by (10) and (11) to study the remaining six unknowns V_k and $\mathcal{E}_k$. It is also easy to obtain from (7) and (8) the constant of motion

$$\phi_a + \phi_b + \phi_c + \frac{1}{3\epsilon_d + 2}(N - 1) = \phi_0, \quad (12)$$

where the constant ϕ_0 must be zero for a neutral plasma.

For given initial values and the external field $\mathcal{E}_{0j}(\tau)$, the mathematically simple ordinary differential equations (7)–(12) can be integrated [10] numerically to yield exact nonlinear-wave solutions. The constant of motion given by (12) is useful for verifying the numerical results. Depending on the initial conditions and the behavior of the external electric field, waves with many different symmetries and temporal behaviors can be obtained. Similar exact solutions have been obtained numerically for cylindrical surface plasma waves [13].

It is of interest to look at the linear and weakly nonlinear limits. Here, one finds that the variable sets (N, V_i, ϕ_i) and (V_k, $\mathcal{E}_k$) are the eigenfunctions of independent *linear modes*. The former set represents volume modes corresponding to the ordinary electron plasma waves. The other set, which involves no density perturbation, describes surface electron plasma modes in spherical geometry. We note that the external field $\mathcal{E}_{0j}$ is coupled only to the surface modes. It is also easily seen from (10) and (11) that if $\mathcal{E}_{0j}$ oscillates at the fundamental surface wave frequency $(1 + 2\epsilon_d)^{-\frac{1}{2}}\omega_p$ (or in general, at the frequency $[\ell/(\ell + (1 + \ell)\epsilon_d)]^{\frac{1}{2}}\omega_p$, where $\ell = 1, 2, 3\ldots$, depending on the geometry of the linear mode), resonant excitation of the surface modes can occur. However, the equations also indicate that the nonlinear terms do not couple the different modes resonantly, so that the familiar nonlinear phenomenon of three-wave or four-wave coupling does not occur in the present class of solutions.

In order to recover the results of Ref. [10], where no external field was included and a particular symmetry was chosen, we need to set $\mathcal{E}_{0j} = 0$ and identify $\frac{1}{3}(\phi_a - \phi_c)$ with Φ, V_a, and V_b with W, and V_c with V, where Φ, W, and V are defined in Ref. [10].

To conclude, we have derived exact nonlinear ordinary differential equations governing the propagation and evolution of electron plasma waves in a plasma sphere bounded by a dielectric containing an external electric field of arbitrary time dependence. Because of the method of separation of variables used, the corresponding solutions are of a special class. Our equations show that only the surface modes can be driven by the external field. Although the other modes are nonlinearly coupled to, and can in principle feed energy into the surface modes, they cannot be driven by the latter or by the external field. It should, however, be emphasized that these conclusions may apply only to the class of solutions presented here and that more general solutions (e.g., nonseparable ones) may behave differently.

Our results may be of interest to surface-wave generated plasmas, control of plasmas for material processing

[2,4,5], and for verifying approximation and numerical methods. Since the evolution equations have been obtained without making use of perturbations and truncations of any kind, when appropriate dissipation is included, the system can provide another mathematically exact model for investigating linear and nonlinear wave instabilities, saturation, nonlinear states, as well as deterministic chaos [12]. On the other hand, if we consider the plasma as a spherical nonlinear dielectric and the surrounding medium as vacuum or air, we arrive at a more exact model of a nonlinear dust or microparticle, which may be useful for investigating light scattering off nonlinear dust particles and the resulting optical bistability problem [11].

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